Midterm 01

Linear Models – Fall 2015

2015-10-12

I. This first question studies a dataset of 100 people with variables on height, weight, and gender. Below are four (slightly truncated) regression tables as well as the first 4 rows of data. These are followed by 5 questions that you need to answer.

```
> summary(out1 <- lm(weight ~ height, data=df))

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -90.0170    18.1786  -4.952 3.07e-06 ***
height        0.9393     0.1092   8.605 1.28e-13 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 8.259 on 98 degrees of freedom
Multiple R-squared:  0.4304,  Adjusted R-squared:  0.4245
F-statistic: 74.04 on 1 and 98 DF,  p-value: 1.28e-13

> summary(out2 <- lm(weight ~ height + gender, data=df))

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -58.9421    22.9359  -2.570   0.0117 *
height        0.7386     0.1419   5.204 1.09e-06 ***
genderM       4.6336     2.1478   2.157   0.0334 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 8.11 on 97 degrees of freedom
Multiple R-squared:  0.4564,  Adjusted R-squared:  0.4452
F-statistic: 40.73 on 2 and 97 DF,  p-value: 1.444e-13

> summary(out3 <- lm(weight ~ height:gender, data=df))

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)    -56.8507    23.5363  -2.415   0.0176 *
height:genderF   0.7256     0.1458   4.977 2.81e-06 ***
height:genderM   0.7534     0.1374   5.483 3.32e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 8.109 on 97 degrees of freedom
Multiple R-squared:  0.4565,  Adjusted R-squared:  0.4453
F-statistic: 40.74 on 2 and 97 DF,  p-value: 1.431e-13
```

```
> summary(out4 <- lm(weight ~ height*gender, data=df))

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)   -56.05426   31.36208  -1.787 0.077041 .
height          0.72066    0.19418   3.711 0.000346 ***
genderM        -1.84814   47.77475  -0.039 0.969222
height:genderM  0.03887    0.28621   0.136 0.892255
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 8.151 on 96 degrees of freedom
Multiple R-squared:  0.4565,  Adjusted R-squared:  0.4396
F-statistic: 26.88 on 3 and 96 DF,  p-value: 1.045e-12

> head(df,4)
  weight height gender
1     62    161      F
2     82    170      M
3     83    174      M
4     71    173      F
```

1. Describe how you would interpret the point estimate for `genderM` in model `out2`. Construct a two-sided, 95.44% condfidence interval for this parameter (i.e., critical value is 2).

2. Write down the first four rows of the model matrix used in models `out3` and `out4`.

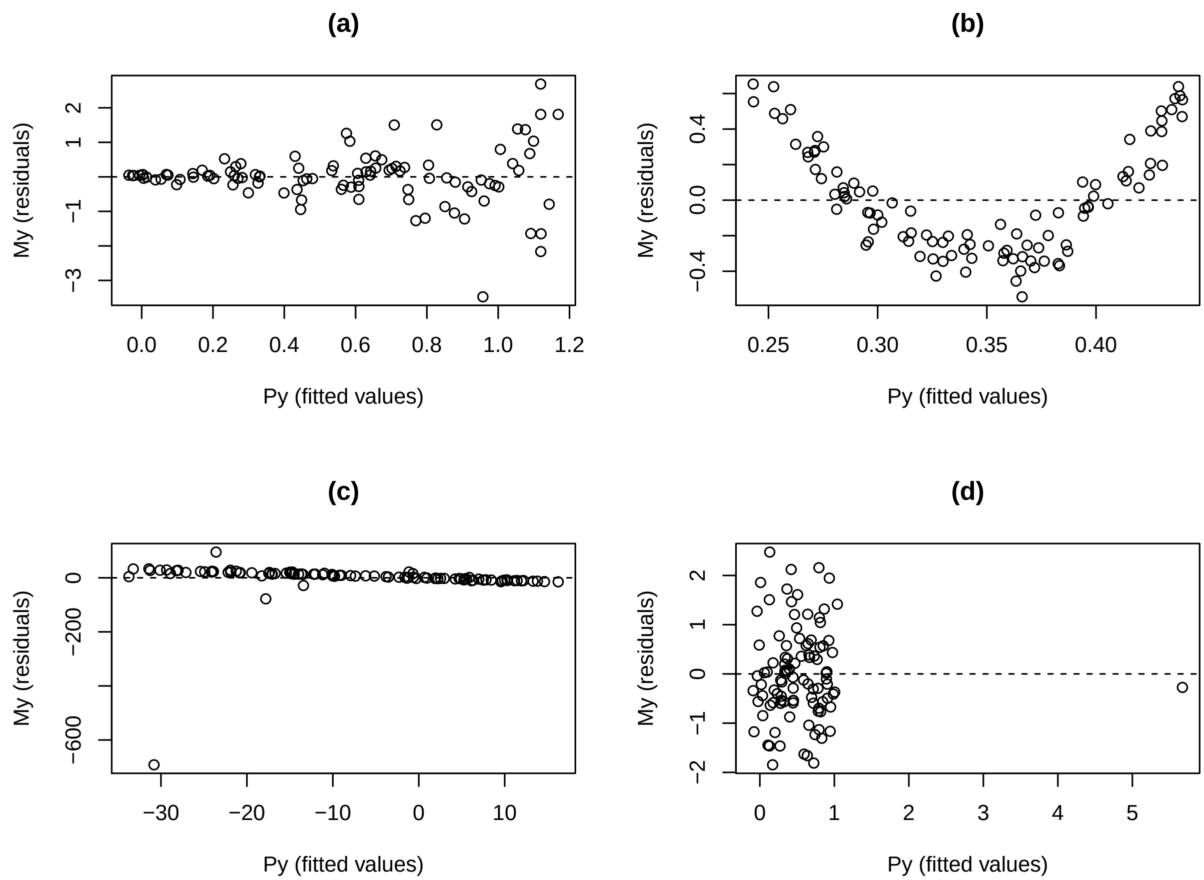
3. Interpret the estimates for `height:genderF` and `height:genderM` in model `out3`. Is there any evidence that these two quantities are different?

4. Explain how both `genderM` and `height:genderM` could not be significant in model `out4` even though they are in models `out2` and `out3`.

5. Construct an F-statistic to simultaneously test whether `genderM` and `height:genderM` are both equal to zero. You can leave the equation unsimplified.

II. Respond to the following short answer questions. Please write in full, clear sentences. Only use mathematical symbols when absolutely required.

1. Below are four plots from linear regression simulations. Describe what violations of the classical linear model assumptions (if any) are violated. Do you expect the estimated $\widehat{\beta}$ to be unbiased?



2. Give a geometric interpretation of why the projection matrix and annihilator matrix are idempotent ($P = P^2$ and $M = M^2$).

3. If $u_i \sim_{i.i.d.} \mathcal{N}(0, 1)$ for i between 1 and n, what is the distribution of $u^t P u$ where P is the projection matrix of an n-by-p matrix X.

4. Construct an example where strict exogeneity is violated but weak exogeneity is not.

5. What is the conceptual difference between prediction intervals and confidence intervals?

6. In what way is the Gauss-Markov theorem (BLUE) stronger than the Cramér–Rao bound?