I. This first question studies a dataset of 100 people with variables on height, weight, and gender. Below are four (slightly truncated) regression tables as well as the first 4 rows of data. These are followed by 5 questions that you need to answer.

```r
> summary(out1 <- lm(weight ~ height, data=df))

Coefficients:
  Estimate  Std. Error   t value   Pr(>|t|)
(Intercept) -90.0170     18.1786   -4.952 3.07e-06 ***
height        0.9393      0.1092    8.605 1.28e-13 ***
---
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 8.259 on 98 degrees of freedom
Multiple R-squared: 0.4304, Adjusted R-squared: 0.4245
F-statistic: 74.04 on 1 and 98 DF,  p-value: 1.28e-13

> summary(out2 <- lm(weight ~ height + gender, data=df))

Coefficients:
  Estimate  Std. Error  t value  Pr(>|t|)
(Intercept) -58.9421     22.9359   -2.570  0.0117 *
genderM       4.6336      2.1478    2.157  0.0334 *
height        0.7386      0.1419    5.204  1.09e-06 ***
---
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 8.11  on 97 degrees of freedom
Multiple R-squared: 0.4564, Adjusted R-squared: 0.4452
F-statistic: 40.73 on 2 and 97 DF,  p-value: 1.444e-13

> summary(out3 <- lm(weight ~ height:gender, data=df))

Coefficients:
  Estimate  Std. Error  t value  Pr(>|t|)
(Intercept) -56.8507     23.5363   -2.415  0.0176 *
height:genderF  0.7256      0.1458    4.977  2.81e-06 ***
height:genderM  0.7534      0.1374    5.483  3.32e-07 ***
---
Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 8.109 on 97 degrees of freedom
Multiple R-squared: 0.4565, Adjusted R-squared: 0.4453
F-statistic: 40.74 on 2 and 97 DF,  p-value: 1.431e-13
```
> summary(out4 <- lm(weight ~ height*gender, data=df))

Coefficients:
  Estimate Std. Error t value Pr(>|t|)
(Intercept)  -56.05426   31.36208  -1.787  0.077041 .
height        0.72066    0.19418   3.711  0.000346 ***
genderM      -1.84814   47.77475  -0.039  0.969222
height:genderM  0.03887   0.28621   0.136  0.892255
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 8.151 on 96 degrees of freedom
Multiple R-squared:  0.4565,  Adjusted R-squared:  0.4396
F-statistic: 26.88 on 3 and 96 DF,  p-value: 1.045e-12

> head(df,4)

   weight height gender
1     62     161    F
2     82     170    M
3     83     174    M
4     71     173    F

1. Describe how you would interpret the point estimate for genderM in model out2. Construct a two-sided, 95.44% confidence interval for this parameter (i.e., critical value is 2).

2. Write down the first four rows of the model matrix used in models out3 and out4.

3. Interpret the estimates for height:genderF and height:genderM in model out3. Is there any evidence that these two quantities are different?

4. Explain how both genderM and height:genderM could not be significant in model out4 even though they are in models out2 and out3.

5. Construct an F-statistic to simultaneously test whether genderM and height:genderM are both equal to zero. You can leave the equation unsimplified.
II. Respond to the following short answer questions. Please write in full, clear sentences. Only use mathematical symbols when absolutely required.

1. Below are four plots from linear regression simulations. Describe what violations of the classical linear model assumptions (if any) are violated. Do you expect the estimated $\hat{\beta}$ to be unbiased?

   ![Plots](a,b,c,d)

2. Give a geometric interpretation of why the projection matrix and annihilator matrix are idempotent ($P = P^2$ and $M = M^2$).

3. If $u_i \sim_{i.i.d.} \mathcal{N}(0, 1)$ for $i$ between 1 and $n$, what is the distribution of $u^t Pu$ where $P$ is the projection matrix of an $n$-by-$p$ matrix $X$.

4. Construct an example where strict exogeneity is violated but weak exogeneity is not.

5. What is the conceptual difference between prediction intervals and confidence intervals?

6. In what way is the Gauss-Markov theorem (BLUE) stronger than the Cramér–Rao bound?